Proof of Rule WF2

The proof uses the following tautologies, where F, G, and H are arbitrary temporal formulas.

T1. $\vdash \Box(F \Rightarrow G) \;\Rightarrow\; (\Diamond F \Rightarrow \Diamond G)$

T2. $\vdash \Box(F \Rightarrow G) \land \Box\Diamond F \;\Rightarrow\; \Box\Diamond G$

T3. $\vdash \Box(F \Rightarrow G \lor H) \land \Box F \;\Rightarrow\; \Box G \lor \Diamond H$

Let EB equal $\overline{\text{ENABLED } \langle B\rangle_w}$.
Here's the rule:

$$\frac{\begin{array}{l} \langle \mathcal{N} \land C\rangle_v \Rightarrow \langle \overline{B}\rangle_{\overline{w}} \\ P \land P' \land \langle \mathcal{N} \land A\rangle_v \land EB \Rightarrow C \\ P \land EB \Rightarrow \text{ENABLED } \langle A\rangle_v \\ \Box[\mathcal{N} \land \neg C]_v \land \mathrm{WF}_v(A) \land \Box EB \Rightarrow \Diamond\Box P \end{array}}{\mathrm{WF}_v(A) \Rightarrow \overline{\mathrm{WF}_w(B)}}$$

Here's the proof.

ASSUME: A1. $\Box(\langle \mathcal{N} \land C\rangle_v \Rightarrow \langle \overline{B}\rangle_{\overline{w}})$

A2. $\Box(P \land P' \land \langle \mathcal{N} \land A\rangle_v \land EB \Rightarrow C)$

A3. $\Box(P \land EB \Rightarrow \text{ENABLED } \langle A\rangle_v)$

A4. $\Box(\Box[\mathcal{N} \land \neg C]_v \land \mathrm{WF}_v(A) \land \Box EB \Rightarrow \Diamond\Box P)$

PROVE: $\Box[\mathcal{N}]_v \land \mathrm{WF}_v(A) \;\Rightarrow\; \overline{\mathrm{WF}_w(B)}$

1. SUFFICES ASSUME: 1. $\Box[\mathcal{N}]_v$
   2. $\mathrm{WF}_v(A)$
   3. $\Diamond\Box EB$

   PROVE: $\Diamond\langle \overline{B}\rangle_{\overline{w}}$

   PROOF: Since $\overline{\mathrm{WF}_w(B)}$ is equivalent to $\Diamond\Box EB \;\Rightarrow\; \Box\Diamond\langle \overline{B}\rangle_{\overline{w}}$, it suffices to prove $\Box\Diamond\langle \overline{B}\rangle_{\overline{w}}$ under these assumptions. The result then follows from the rule $F \Vdash \Box F$, since the assumptions are all $\Box$ formulas.

2. $\Box[\mathcal{N} \land \neg C]_v \lor \Diamond\langle \mathcal{N} \land C\rangle_v$

   2.1. $[\mathcal{N}]_v \;\Rightarrow\; [\mathcal{N} \land \neg C]_v \lor \langle \mathcal{N} \land C\rangle_v$

   PROOF: This follows from the definitions of $[\ldots]_v$ and $\langle \ldots\rangle_v$ and the propositional logic tautology:

   $$(U \lor V) \;\Rightarrow\; ((U \land \neg W) \lor V) \lor (U \land W \land \neg V)$$

   2.2. Q.E.D.

   By step 2.1, T3, and the rule $F \Vdash \Box F$.

3. $\Diamond\langle \mathcal{N} \land C\rangle_v \Rightarrow \Diamond\langle \overline{B}\rangle_{\overline{w}}$

PROOF: By T1 and assumption A1.

4. ASSUME: $\Box[\mathcal{N} \land \neg C]_v$
   PROVE: FALSE

   4.1. $\Box[\mathcal{N} \land \neg C]_v \land \mathrm{WF}_v(A) \land \Diamond\Box EB \Rightarrow \Diamond\Box P$

   PROOF: A4 and T1 imply

   $$\Diamond\Box[\mathcal{N} \land \neg C]_v \land \Diamond\mathrm{WF}_v(A) \land \Diamond\Box EB \Rightarrow \Diamond\Diamond\Box P$$

   Step 4.1 follows from this and the tautologies $F \Rightarrow \Diamond F$ and $\Diamond\Diamond F \equiv \Diamond F$.

   4.2. $\Diamond\Box P$

   PROOF: By 4.1, the steps 1 and 4 assumptions.

   4.3. $\Box(P \land P' \land \langle\mathcal{N} \land A\rangle_v \land EB \Rightarrow \langle\mathcal{N} \land C\rangle_v)$

   PROOF: By A2, since $\langle\mathcal{N} \land A\rangle_v \land C$ implies $\langle\mathcal{N} \land C\rangle_v$ by definition of $\langle \ldots \rangle_v$.

5. Q.E.D.

   PROOF: By steps 2, 3, and 4. $(F \Rightarrow G) \Vdash (\Box\Diamond F \Rightarrow \Box\Diamond G)$.

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