An Informal Proof of Deadlock Freedom

0 **Theorem** The 2-process 1-bit algorithm satisfies *DeadlockFree*

1. It suffices to assume that $T0 \lor T1$ is true at some time t_1 and $\neg Success$ is true at all times $t \geq t_1$, and to obtain a contradiction.

   PROOF: By definition of deadlock freedom.

2. $T0$ is false at every time $t \geq t_1$.

   ...

3. $T1$ is false at time t_1.

   3.1. It suffices to assume that $T1$ is true at time t_1 and obtain a contradiction.

   PROOF: Obvious.

   3.2. $T1$ is true at all times $t \geq t_1$.

   PROOF: By the step 1 assumption, $\neg InCS(1)$ (which is implied by $\neg Success$) is true for all times $t \geq t_1$. From the code and the step 3.1 assumption, this implies that $T1$ is true at all times $t \geq t_1$.

   3.3. There is some time $t_2 \geq t_1$ such that $\neg x[0]$ is true for all times $t \geq t_2$.

   PROOF: By the code and fairness, step 2 implies that process 0 reaches *ncs* and remains there forever at some time $t_2 \geq t_1$,

   3.4. $T_1 \land \neg x[0]$ is true for all times $t \geq t_2$

   PROOF: By steps 3.2 and 3.3.

   3.5. Q.E.D.

   PROOF: Step 3.4, the code, and fairness imply that process 1 reaches $e2$ at some time $t_3 \geq t_2$, which by fairness and 3.4 implies that process 1 reaches its critical section at some time $t_4 > t_3$. Since $t_4 \geq t_1$, this contradicts the assumption from step 1 that $\neg Success$ is true for all $t \geq t_1$.

4. Q.E.D.

   PROOF: Steps 2 and 3 and the step 1 assumption.

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