Answer

Let Π be the process's next-state action and let A_1, …, A_n be the process's actions. We prove that a behavior σ is weakly fair for Π iff it is weakly fair for all the A_i. The proof is long, but it involves simple expansion of the definitions to state, for each step, what must be proved.

$\langle 1 \rangle 1$. $\Pi \equiv A_1 \lor \ldots \lor A_n$

PROOF: This is the definition of a process's next-state action.

$\langle 1 \rangle 2$. Π is enabled iff some A_i is enabled

PROOF: By $\langle 1 \rangle 1$ and the definition of enabled.

$\langle 1 \rangle 3$. ASSUME: 1. σ is a behavior that is weakly fair for Π
2. $i \in 1 \,..\, n$

PROVE: σ is weakly fair for A_i

$\langle 2 \rangle 1$. σ does not end in a state in which A_i is enabled

PROOF: Assumption $\langle 1 \rangle 3.1$ and the definition of weakly fair implies σ does not end in a state in which Π is enabled, which by $\langle 1 \rangle 2$ implies $\langle 2 \rangle 1$.

$\langle 2 \rangle 2$. σ does not contain an infinite suffix τ such that A_i is enabled in every state of τ and τ contains no A_i step.

$\langle 3 \rangle 1$. SUFFICES ASSUME: τ is an infinite suffix of σ with A_i enabled in every state

PROVE: τ contains an A_i step.

PROOF: By simple logic.

$\langle 3 \rangle 2$. Π is enabled in every state of τ

PROOF: By the step $\langle 3 \rangle 1$ assumption and $\langle 1 \rangle 2$.

$\langle 3 \rangle 3$. τ contains a Π step.

PROOF: By $\langle 3 \rangle 2$, assumption $\langle 1 \rangle 3.1$, and the definition of weakly fair.

$\langle 3 \rangle 4$. A Π step starting in a state with A_i enabled is an A_i step.

PROOF: By $\langle 1 \rangle 1$, since, for any j, action A_j is enabled only if control in the process is at its label.

$\langle 3 \rangle 5$. Q.E.D.

PROOF: By $\langle 3 \rangle 1$, $\langle 3 \rangle 3$, and $\langle 3 \rangle 4$.

$\langle 2 \rangle 3$. Q.E.D.

PROOF: By $\langle 2 \rangle 1$, $\langle 2 \rangle 2$, and the definition of of weakly fair.

$\langle 1 \rangle 4$. ASSUME: σ is a behavior that is weakly fair for A_i, for all $i \in 1 \,..\, n$.

PROVE: σ is weakly fair for Π

$\langle 2 \rangle 1$. σ does not end in a state in which Π is enabled

PROOF: By $\langle 1 \rangle 2$, Π enabled implies A_i is enabled for some i, and the $\langle 1 \rangle 4$ assumption and the definition of weakly fair implies that σ does not end in any state in which an A_i is enabled.

⟨2⟩2. σ does not contain an infinite suffix τ such that Π is enabled in every state of τ and τ contains no Π step.

⟨3⟩1. SUFFICES ASSUME: 1. τ is an infinite suffix of σ with Π enabled in every state.
2. τ contains no Π step.
PROVE: FALSE

This is a proof by contradiction.

PROOF: Simple logic.

⟨3⟩2. Choose i such that the process's control is at the label of A_i in the first state of τ.

PROOF: By assumption ⟨3⟩1.1, since Π enabled implies control is at one of its labels.

⟨3⟩3. Control is at the label of A_i in every state of τ.

PROOF: By ⟨3⟩2 and assumption ⟨3⟩1.2, since control in the process can be changed only by a Π step.

⟨3⟩4. A_i is enabled in every state of τ.

PROOF: By ⟨3⟩3 and assumption ⟨3⟩1.1.

⟨3⟩5. τ contains an A_i step.

PROOF: By ⟨3⟩4, the step ⟨1⟩4 assumption, and the definition of weakly fair.

⟨3⟩6. Q.E.D.

PROOF: ⟨3⟩5 and ⟨1⟩1 contradict assumption ⟨3⟩1.2.

⟨2⟩3. Q.E.D.

PROOF: By ⟨2⟩1, ⟨2⟩2, and the definition of of weakly fair.

⟨1⟩5. Q.E.D.

PROOF: By ⟨1⟩3 and ⟨1⟩4.

CLOSE