Explanation of the Proof

We must show that to prove

$$Inv \wedge (\exists\, i \in \{0,1\} : e1(i) \vee e2(i) \vee CS(i) \vee Rest(i)) \;\Rightarrow\; Inv'$$

it suffices to prove steps 1–4 of the proof. Here is the argument:

1. It suffices to prove

   $$(\exists\, i \in \{0,1\} : Inv \wedge (e1(i) \vee e2(i) \vee CS(i) \vee Rest(i))) \;\Rightarrow\; Inv'$$

   PROOF: Since i does not occur free in Inv, the formulas

   $$Inv \wedge (\exists i \in \{0,1\} : \ldots) \quad \text{and} \quad (\exists i \in \{0,1\} : Inv \wedge \ldots)$$

   are equivalent.

2. It suffices to prove

   $$(i \in \{0,1\}) \wedge Inv \wedge (e1(i) \vee e2(i) \vee CS(i) \vee Rest(i)) \;\Rightarrow\; Inv'$$

   PROOF: For any P and Q, if i does not occur free in Q, then to prove $(\exists i \in S : P(i)) \Rightarrow Q$, it suffices to prove $(i \in S) \wedge P(i) \Rightarrow Q$.

3. $(i \in \{0,1\}) \wedge Inv \wedge (e1(i) \vee e2(i) \vee CS(i) \vee Rest(i))$ is equivalent to

   $$\begin{array}{l} \vee\ (i \in \{0,1\}) \wedge Inv \wedge e1(i) \\ \vee\ (i \in \{0,1\}) \wedge Inv \wedge e2(i) \\ \vee\ (i \in \{0,1\}) \wedge Inv \wedge CS(i) \\ \vee\ (i \in \{0,1\}) \wedge Inv \wedge Rest(i) \end{array}$$

   PROOF: By propositional logic.

4. Q.E.D.

   PROOF: By steps 2 and 3, since $(P_1 \vee \ldots \vee P_k) \Rightarrow Q$ is equivalent to $(P_1 \Rightarrow Q) \wedge \ldots \wedge (P_k \Rightarrow Q)$.

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